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# Enhancing differential GPS using regional ionospheric error models

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**Abstract.** Distance-dependent errors due to ionospheric refraction complicate ambiguity resolution and limit the accuracy attainable in GPS baseline determination. This paper presents an approach for modelling these errors from the observations of several permanent GPS-stations surrounding the area of interest. Regional ionospheric correction models are produced epoch-by-epoch and satellite-by-satellite. It is shown that after their application, observation residuals, fractional parts of estimated ambiguities, and single-frequency coordinate errors are greatly reduced, thus improving ambiguity resolution even under disturbed ionospheric conditions, with short observation sessions, in the case of baseline lengths of more than 10 km, and with single-frequency and dual-frequency data.

## 1 Introduction

Differential GPS based on carrier phase observations can provide accuracies of coordinate differences in the order of ±(0.5 cm + 1 ppm) or better if the carrier phase ambiguities could be resolved and fixed (*Seeber 1993*). With the ambiguities being fixed, the accuracy increases only very slowly with extended observation periods. In the case of single-frequency observations, ionospheric refraction can cause an additional error up to some ppm of the baseline length. Dual-frequency observations can effectively be corrected for ionospheric refraction by forming the ionosphere-free linear combination.

Ionospheric conditions can affect ambiguity resolution even of dual-frequency data, because the ionosphere-free linear combination and other linear combinations showing little ionospheric effects have the property of rather short wavelengths which complicates ambiguity resolution. Differential effects of ionospheric refraction are mainly caused by large-scale gradients of the vertical electron content and by ionospheric disturbances (*Wanninger 1993a,b*). Due to these distance-dependent errors, ambiguity resolution gets the more difficult, the longer the baseline is. Ionospheric errors can often be reduced by averaging over long observation periods, i.e. the longer the baseline, the more measurements are required to reliably resolve the ambiguities. With distances up to a few kilometers, ionospheric effects do not complicate ambiguity resolution. With baseline lengths of more than 10 km, however, ionospheric disturbances may prevent ambiguity resolution with single-frequency data or even with dual-frequency data.

The effects of orbit errors also depend on the baseline length but they are usually smaller in size. Modelling errors of the tropospheric refraction are rather a problem with height differences then with distance. Multipath effects and random observation errors are station and receiver dependent and therefore independent of baseline length.

In some areas a dense network of permanent GPS tracking stations already exists or will be installed in the near future. GPS stations at known locations will allow the estimation of differential ionospheric errors between these stations. This information can than be used to predict (interpolate) corrections for this kind of errors for baselines between any two stations in the area. This technique of estimation, prediction, and correction of the distance-dependent ionospheric errors enhances differential GPS. It enables faster ambiguity resolution for distances of more than 10 km and reduces errors in coordinate differences derived from single-frequency observations.

Earlier investigations made use of one dual-frequency receiver placed in the center of a network to estimate ionospheric delays for single-frequency receivers in the vicinity and thus to reduce baseline coordinate errors (*Georgiadou and Kleusberg 1988*). This work was extended to include any number of dual-frequency receivers (*Wild et al. 1989*) and to improve ambiguity resolution on long baselines (*Mervart et al. 1994*). These approaches make use of one ionospheric model estimated for an observation session of several hours and used for all satellites observed in the session. Thus, the model cannot remove small-scale or medium-scale structures of ionospheric refraction.

This method was extended by *Webster and Kleusberg (1992)* to provide epoch-by-epoch and satellite-by-satellite ionospheric corrections. The ionospheric delays of a station equipped with a single-frequency receiver are estimated from interpolation of ionospheric delay observations of three surrounding monitor stations using the intersection points of the GPS signal paths with an ionospheric single-layer model at a height of 350 km. The problem of ambiguities in the ionospheric delays derived from dual-frequency phase data is overcome by assuming that the ambiguity differences between the monitor stations are, on average, equal to zero.

In this research dual-frequency carrier phase ambiguities are resolved and fixed in the network of monitor stations, thus yielding differential ionospheric delays in the most accurate GPS mode. Correction values are then determined on the level of double differences epoch-by-epoch and satellite-by-satellite from interpolation of ionospheric delay values of the monitor stations using their coordinates and approximate coordinates of the new stations.

## 1.1 The Double Difference Observable

By forming double differences (between-receiver and between-satellite differences) of the carrier phase observations, several error sources are removed (clock errors, hardware delays) or at least greatly reduced (ionospheric refraction, tropospheric refraction, orbit errors, phase center offsets). Other errors are increased (multipath, observation noise). Only on this level of differencing, the carrier phase ambiguities can be determined to integer values and can therefore be removed. The double difference observation processing mode is one of the most used techniques of operational GPS softwares. Data processing based on non-differenced observations uses comparable techniques to solve for integer ambiguities (*Goad 1985*).

The differencing operation is performed in such a way that the observations at a base station (one of the two stations of a baseline, or in general one of the simultaneously observed stations) and to a base satellite (selected arbitrarily, or by observation length, or by elevation angle) are common to every double difference observable.

The double difference carrier phase observation equation can be written as follows, omitting indices for the base station, a second station, the base satellite, and a second satellite:

$$\nabla\Delta\Phi = \nabla\Delta R + \lambda \cdot \nabla\Delta N + \nabla\Delta i + \nabla\Delta\epsilon \tag{1}$$

where

$\nabla\Delta\Phi$ : double difference carrier phase observable (in length units)
$\nabla\Delta R$ : geometric double difference range
$\lambda$ : carrier wavelength
$\nabla\Delta N$ : integer double difference carrier phase ambiguity
$\nabla\Delta i$ : double difference range error due to ionospheric refraction
$\nabla\Delta\epsilon$ : double difference observation noise and remaining unmodelled effects.

In equation (1), we distinguish between the ionospheric error $\nabla\Delta i$ and other unmodelled effects $\nabla\Delta\epsilon$. In our approach of enhanced differential GPS we determine $\nabla\Delta i$ for known baselines between permanent GPS tracking stations and predict corrections for this kind of error for other baselines (stations) in the area.

Those unmodelled effects, which influence the observables independently of the signal's frequency (orbit errors, tropospheric refraction), do not affect the determination of the double difference ionospheric error $\nabla\Delta i$. However, frequency-dependent errors (multipath effects, observation noise) affect its determination.

The accurate estimation of $\nabla\Delta i$ can only be achieved if the carrier phase ambiguities are determined to integer values. This necessary pre-processing step imposes limitations on the application of the enhanced differential GPS correction technique: if the permanent GPS stations are some 50 km apart, at least an hour of observations is needed to resolve the ambiguities of a newly risen satellite, i.e. the algorithm cannot be applied in (near-)realtime. Moreover, it limits the distance between the permanent stations. If they are more than some 50 km apart, dual-frequency ambiguity resolution cannot always be guaranteed for low and short satellite passes.

# 2 The Algorithm

## 2.1 Overview

The ionospheric model is based on the double difference residuals of the ionospheric (geometry-free) linear combination $L_i$ of the phase observations. Error sources affecting the ionospheric observable (multipath effects, observation noise) are considered as noise.

In a first processing step (compare Figure 1) ambiguity resolution and fixing has to be performed for the network of permanent GPS stations. Although their distances may be of the order of 50 km, ambiguity resolution is simplified because the baseline coordinates are known, dual-frequency receivers are employed and long observation periods can be used.

As a result, we obtain double difference residuals in $L_1$ and $L_2$ which can than be transformed to double difference residuals of the ionospheric signal (see subsection 2.2). If three permanent stations are available, the interpolation of correction values is performed by a linear interpolation algorithm as described in subsection 2.3. If more permanent stations are available, either the best group of three stations (for example the three closest surrounding stations) should be selected or several correction values from sets of three stations can be computed and averaged. The main advantage of more than

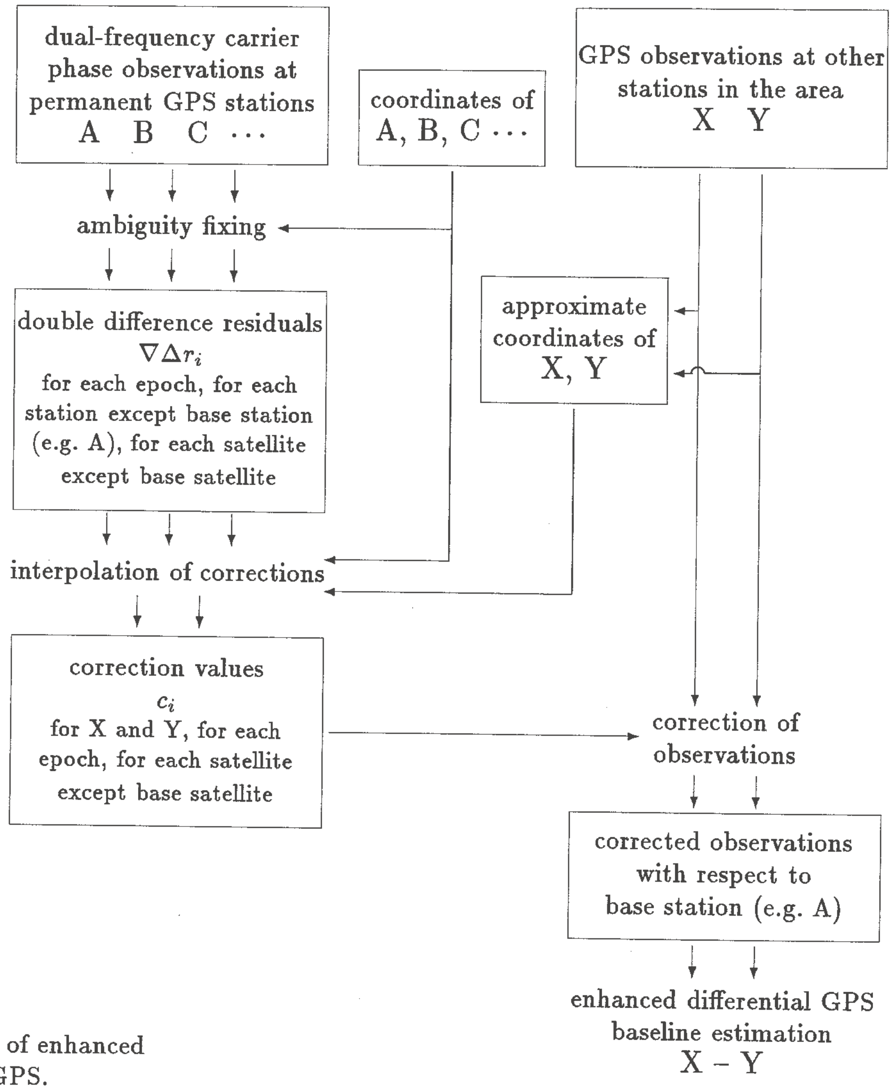


Figure 1:
The concept of enhanced differential GPS.

three permanent stations lies in the ability to determine correction values even if only some (but at least three) permanent stations including the base station could provide observations.

The correction values are interpolated epoch by epoch, i.e. in each observation epoch a new ionospheric model is created. It is valid for just this particular epoch.

In a further processing step, the ionospheric correction value $c_i$ is transformed to yield corrections for the original phase observations $L_1$, $L_2$ or also for the code observations $C_1$ and $C_2$ (see subsection 2.4). The observations of the base station and of the base satellite need not to be modified because their correction values are zero.

Since double difference corrections are applied to non-differenced observations, a further baseline processing must only be performed between stations whose observations have been manipulated by correction values based on identical error models. Manipulated data must not be combined with original observations with the exception of the observations of the base station.

In an operational mode, ambiguity resolution in a regional network of several reference stations surrounding a number of new stations consists of the following processing steps: (a) ambiguity resolution in the network of reference stations for an observation period of several hours or a complete day, (b) estimation of correction values as described in this paper, (c) modification of the observations of all stations with the exception of the base station, and (d) improved ambiguity resolution for all baselines in the network.

## 2.2 Ionospheric Errors

In order to estimate the ionospheric error and also in order to be able to correct all observables for this error, it will be shown how it can be computed from $L_1$ and $L_2$ phase residuals.

The ionospheric refractive phase index $n_{PH}$ for frequencies above 100 MHz is given by (see e.g. *Seeber*

*1993*):

$$n_{PH} = 1 - 40.3\,\frac{N_e}{f^2} \;\pm\; \ldots \qquad (2)$$

where

$f$ : signal frequency [Hz]
$N_e$ : density of free electrons [$m^{-3}$].

Higher order terms can be neglected in our application. The ionospheric range error $\delta R$ is obtained by integrating the first order term of the refractive index along the signal path from the satellite $S$ to the receiver $R$

$$\delta R = -40.3\,\frac{1}{f^2}\cdot\int_R^S N_e(s)ds \;=\; -40.3\,\frac{TEC}{f^2}. \qquad (3)$$

The integral over the electron density is called Total (ionospheric) Electron Content (TEC) [$m^{-2}$].

Thus, the geometric distance $R_0$ [m], free of ionospheric effects, equals the measured distance $R$ corrected by the ionospheric error $\delta R$ (ambiguities and other error sources are neglected).

$$R_0 = R - \delta R \qquad (4)$$

Applying equations (3) and (4) to the GPS frequencies $f_1$=1575.42 MHz and $f_2$=1227.60 MHz yields:

$$R_0 = R_1 + 40.3\,\frac{TEC}{f_1^2} \qquad (5)$$

$$R_0 = R_2 + 40.3\,\frac{TEC}{f_2^2}, \qquad (6)$$

where $R_1$ and $R_2$ are the measured distances at GPS frequencies $f_1$ and $f_2$ respectively. The ionosphere-free observable $R_0$ and the Total Electron Content $TEC$ can be determined from these simultaneous dual-frequency measurements. Eliminating $TEC$ we obtain:

$$R_0 = \frac{f_1^2}{f_1^2 - f_2^2}\cdot R_1 - \frac{f_2^2}{f_1^2 - f_2^2}\cdot R_2. \qquad (7)$$

Eliminating $R_0$ we obtain:

$$TEC = \frac{1}{40.3}\,\frac{f_1^2\cdot f_2^2}{f_1^2 - f_2^2}\,(R_1 - R_2). \qquad (8)$$

Equations (7) and (8) can also be applied to observation residuals or observation differences like the double difference carrier phase observations. Let $\nabla\Delta r_1$ and $\nabla\Delta r_2$ [m] be the double difference carrier phase range residuals of $L_1$ and $L_2$ respectively, we can calculate the ionospheric (geometry-free) residual $\nabla\Delta r_i$ [$m^{-2}$] by:

$$\begin{aligned}\nabla\Delta r_i &= \frac{1}{40.3}\,\frac{f_1^2\cdot f_2^2}{f_1^2 - f_2^2}\cdot(\nabla\Delta r_1 - \nabla\Delta r_2) \qquad (9)\\ &= 9.5196\cdot 10^{16}\cdot(\nabla\Delta r_1 - \nabla\Delta r_2)\end{aligned}$$

## 2.3 Interpolation of Corrections

Correction values for the observations at a new station X are predicted from the double difference residuals at the three stations A, B, and C. Three-dimensional vectors consisting of the two-dimensional station position $(\phi,\lambda)$ and the residuals $(\nabla\Delta r_i)$ define a plane (Figure 2) which is described by the right hand side of equation (10). The left hand side contains the three-dimensional vector of the new station X:

$$\begin{pmatrix}\phi^X\\ \lambda^X\\ \nabla\Delta r_i^X\end{pmatrix} = \begin{pmatrix}\phi^A\\ \lambda^A\\ \nabla\Delta r_i^A\end{pmatrix} + \alpha\left[\begin{pmatrix}\phi^B\\ \lambda^B\\ \nabla\Delta r_i^B\end{pmatrix} - \begin{pmatrix}\phi^A\\ \lambda^A\\ \nabla\Delta r_i^A\end{pmatrix}\right] + \beta\left[\begin{pmatrix}\phi^C\\ \lambda^C\\ \nabla\Delta r_i^C\end{pmatrix} - \begin{pmatrix}\phi^A\\ \lambda^A\\ \nabla\Delta r_i^A\end{pmatrix}\right]. \qquad (10)$$

Since the approximate two-dimensional position of the new station X $(\phi^X, \lambda^X)$ can be determined form GPS code observations, three unknowns remain which are calculated from the three equations in (10): the coefficients $\alpha$, $\beta$ and the residual $\nabla\Delta r_i^X$. For static measurements at the station X, the geometric conditions are identical for all epochs and all satellites. Thus, the coefficients $\alpha$ and $\beta$ need to be determined just once by:

$$\alpha = \frac{\frac{\phi^X-\phi^A}{\phi^C-\phi^A} - \frac{\lambda^X-\lambda^A}{\lambda^C-\lambda^A}}{\frac{\phi^B-\phi^A}{\phi^C-\phi^A} - \frac{\lambda^B-\lambda^A}{\lambda^C-\lambda^A}};\quad \beta = \frac{\frac{\phi^X-\phi^A}{\phi^B-\phi^A} - \frac{\lambda^X-\lambda^A}{\lambda^B-\lambda^A}}{\frac{\phi^C-\phi^A}{\phi^B-\phi^A} - \frac{\lambda^C-\lambda^A}{\lambda^B-\lambda^A}}. \qquad (11)$$

The predicted residual is given by the third equation in (10):

$$\begin{aligned}\nabla\Delta r_i^X &= \nabla\Delta r^A + \alpha\cdot(\nabla\Delta r_i^B - \nabla\Delta r_i^A)\\ &\quad + \beta\cdot(\nabla\Delta r_i^C - \nabla\Delta r_i^A). \qquad (12)\end{aligned}$$

This predicted residual is used as correction value

$$c_i^X = \nabla\Delta r_i^X \qquad (13)$$

for the modification of the observations at station X in the next processing step.

In equations (10), (12) and (13) the superscript letter assigns a double difference residual to one station. The indication of the base station and of the satellites (base satellite and second satellite) have been neglected. If, for example, station A was selected to be the base station, all $\nabla\Delta r^A$ values are equal to zero.

## 2.4 Correction of Observations

One double difference correction value $c_i$ is given for each epoch and for each satellite except for the base satellite. It is used to determine the ionospheric effect on the phase observations $L_1$ and $L_2$ [cycles] and also on

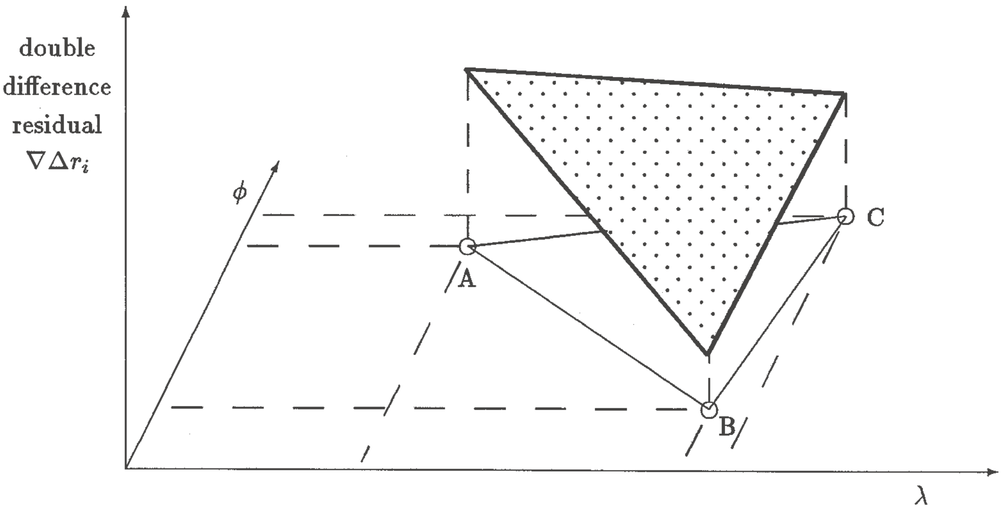


Figure 2: The interpolation model.

the code observations $C_1$ and $C_2$ [m]. Code and phase are affected by first-order ionospheric errors of the same size but in opposite direction (group delay - phase advance, see e.g. *Seeber 1993*). The phase correction values are scaled to units of cycles in order to be in accordance with the RINEX-format (RINEX - Receiver Independent Exchange Format, *Gurtner 1994*). The transformation equations are derived from equations (5) and (6):

$$\begin{bmatrix} c_{L1}\ [cy] \\ c_{L2}\ [cy] \\ c_{C1}\ [m] \\ c_{C2}\ [m] \end{bmatrix} = \begin{bmatrix} -\frac{40.3}{f_1^2}/\lambda_1 \\ -\frac{40.3}{f_2^2}/\lambda_2 \\ \frac{40.3}{f_1^2} \\ \frac{40.3}{f_2^2} \end{bmatrix} [\ c_i\ [m^{-2}]\ ] \qquad (14)$$

$$= \begin{bmatrix} -0.8533 \cdot 10^{-16} \\ -1.0950 \cdot 10^{-16} \\ 0.1624 \cdot 10^{-16} \\ 0.2674 \cdot 10^{-16} \end{bmatrix} [\ c_i\ ] .$$

These corrections can directly be applied to RINEX-formatted observations.

# 3 Examples

Two examples are presented to illustrate the effect of regional ionospheric error models on GPS baseline estimation. In both cases several hours of Trimble SSE GPS tracking data are available for five stations with baseline lengths of 20 to 80 km. The central stations are considered to be unknown stations, the four surrounding stations are used as permanent tracking sites. The algorithm is tested for baselines between the central station and the station which was chosen to be the base station of the double difference error models. Thus, the correction values need to be applied to the observations of the central stations only, because all correction values of the base station are equal to zero. When data of all five stations are available, correction values are calculated for those two triangles, which enclose the central station, and then the average value is taken for the further processing. Ambiguity resolution and baseline estimation were performed with the GEONAP software package (*Wübbena 1989*).

Results are presented in the form of double difference residuals, fractional parts of estimated ambiguities and coordinate errors. This information is shown for the original observations and for the data with applied ionospheric corrections. Double difference residuals are calculated with respect to the dual-frequency baseline solution of the complete observation session. The distributions of double difference residuals are shown for $L_1$ ($\nabla\Delta r_1$), for the widelane linear combination $L_W$ ($\nabla\Delta r_W$), for the ionospheric electron content ($\nabla\Delta r_i$), and for the ionosphere-free linear combination $L_0$ ($\nabla\Delta r_0$).

Double difference ambiguities were determined for 15-minute blocks of observations. In the case of such short observation periods, ambiguity fixing can be difficult due to observation errors and due to the poor estimation of initial baseline coordinates either from code observations (high noise level) or from phase observations with the ambiguities not being fixed (float solution, poor geometry). Since we are just interested in the influence of observation errors on the ambiguity estimation, we kept the initial baseline coordinates fixed to the baseline solution of the complete observation session. Thus, the estimated real number ambiguities for the short 15-minute blocks of observations are closer to the correct integers as they

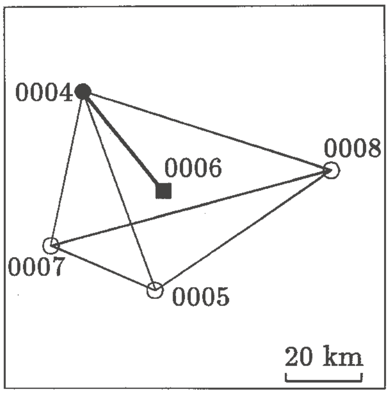


Figure 3a: Permanent GPS stations, baseline 0004-0006 (length: 33.6 km).

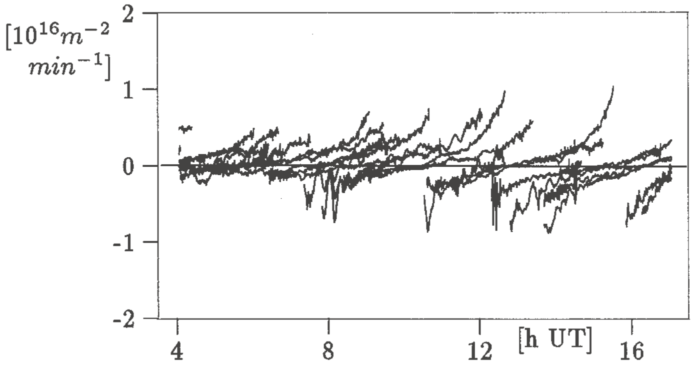


Figure 3b: Ionospheric conditions: Rate of TEC

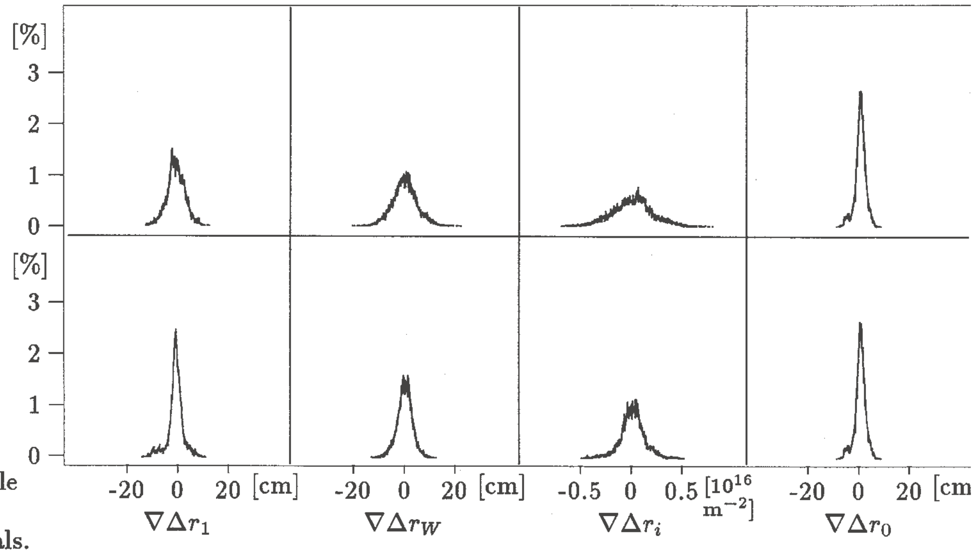


Figure 3c: Distribution of double difference range (or free electron) residuals.

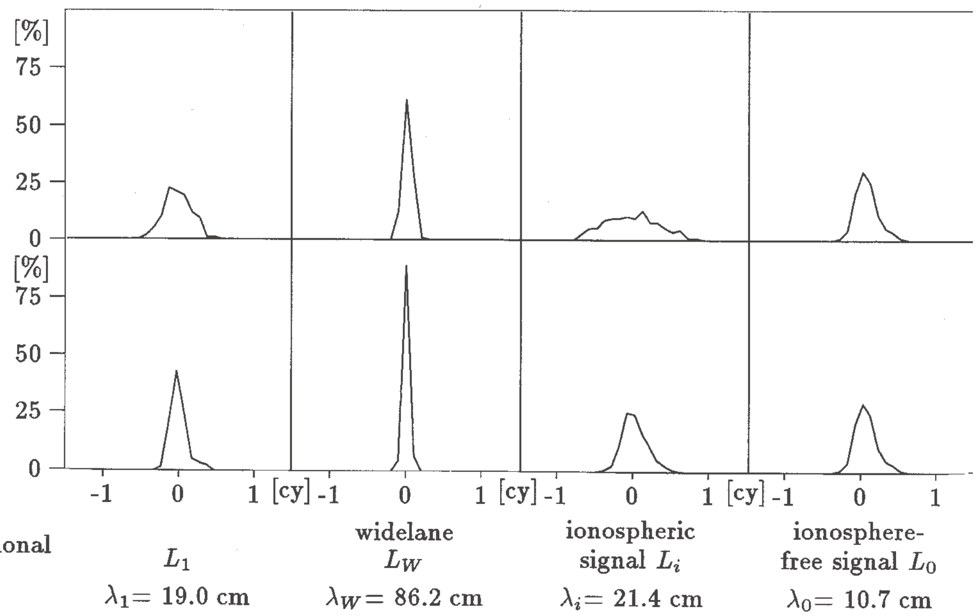


Figure 3d: Distribution of fractional parts of ambiguities (15-min blocks of observations).

would be under realistic processing conditions. However, they perfectly illustrate the effects of regional ionospheric error models on ambiguity resolution. The distribution of fractional parts of the double difference ambiguities are shown graphically for $L_1$, $L_W$, the ionospheric signal $L_i$, and the ionosphere-free signal $L_0$. In the cases of the last two linear combinations, the chosen wavelengths are twice as long as their regular wavelengths and are valid only after fixing the widelane ambiguity (*Wübbena 1989*).

In a last step, coordinate errors were estimated for 15-minute blocks of observations with ambiguities being fixed and compared with the baseline results obtained from the complete data sets.

Range residuals, estimated ambiguities, and coordinate errors of the ionosphere-free linear combination $L_0$ are shown here to provide information on the size and effects of non-ionospheric errors. The $L_0$-observable must be unaffected by ionospheric corrections. Thus, the results presented for $L_0$ must be identical for the original observations and for the ionospheric corrected observations. Nevertheless, they were estimated in order to verify the correct implementation of the algorithm.

## 3.1 Average Mid-latitude Ionospheric Conditions

Observations were selected arbitrarily from the data sets of the already existing dense network of permanent GPS stations in North Germany (Figure 3a). 12 hours of observations (June 17, 1994) were used to test the described algorithm. The ionospheric conditions were analysed with the help of the change of ionospheric refraction over one minute calculated from dual-frequency phase observations (Rate of TEC, *Wanninger 1993a,b*). The Rate of TEC curves of all available satellites reveal that almost undisturbed and thus average mid-latitude ionospheric conditions were present. Some small disturbances occurred at the beginning of a few satellite passes (Figure 3b).

The double difference residuals of the original observations of the 33.6 km baseline (Figure 3c) show that 95% of the ionosphere-free residuals $\nabla\Delta r_0$ are within $\pm 4.8$ cm. The ionospheric double difference residuals are smaller than $\pm 0.39 \cdot 10^{16} \mathrm{m}^{-2}$ (95%) which corresponds to an ionospheric effect on $L_1$-observations of $\pm 6.3$ cm (95%). Thus, in the case of single-frequency observations, ionospheric errors exceed non-ionospheric errors, although only moderate ionospheric conditions were present.

The application of ionospheric corrections considerably reduced existing errors, resulting in ionospheric double difference residuals being smaller than $\pm 0.25 \cdot 10^{16}\ m^{-2}$ (95%).

Similar results were obtained from the distribution of the fractional parts of the 15-minute double difference ambiguities (Figure 3d). 83% of the $L_1$ ambiguities were closer to the correct integer values than 0.2 cycles. The ionospheric correction improved the percentage to 94. Much improvement could be obtained in the case of the ionospheric signal $L_i$, which is of importance to fast ambiguity resolution and kinematic applications. 49% of the ambiguities were closer to the correct integer values than 0.2 cycles before applying the ionospheric correction. Afterwards the percentage exceeded 88. The ionospheric correction considerably improved ambiguity resolution with single-frequency and with dual-frequency observations. Now, fast ambiguity resolution techniques can be used on longer baselines and the observation lengths of baselines between 10 and 40 km can be reduced.

| Signal | original observations | with ionospheric corrections |
|---|---|---|
| $L_1$ | 3.9/2.4/4.1 | 1.8/1.1/3.3 |
| $L_0$ | 1.3/1.1/2.6 | 1.3/1.1/2.6 |

Table 1: Coordinate errors (RMS in cm) in latitude/longitude/height for observation periods of 15 minutes, baseline length 33.6 km, average mid-latitude ionospheric conditions.

Single-frequency and dual-frequency RMS coordinate errors were estimated for the 15-minute blocks of observations (Table 1). The ionospheric corrections reduced the single-frequency coordinate errors in latitude and longitude from 2–4 cm to 1–2 cm in latitude and longitude, and from about 4 cm to about 3 cm in height. Coordinates determined from single-frequency measurements plus ionospheric corrections are almost as accurate as the results from dual-frequency observations.

## 3.2 Disturbed Mid-latitude Ionospheric Conditions

The most common ionospheric disturbances in mid-latitude regions are caused by medium scale travelling ionospheric disturbances (MSTIDs). They mainly occur during daylight hours in winter months in years of maximum solar activity. They complicate ambiguity resolution of single-frequency and dual-frequency data even on baselines shorter than 10 km. Single-frequency coordinate errors can exceed 10 ppm of the baseline length (*Wanninger 1993a*).

An example of strong MSTIDs was found in a 13-hour data set observed in North Germany on March 16, 1993 (Figure 4a). The Rate of TEC curves (Figure 4b) reveal large changes of ionospheric refraction with the typical MSTID-periods of 10 to 20 minutes. Ionospheric corrections obtained from a single ionospheric model estimated for an observation session of several hours and used for all satellites observed cannot account for these medium-scale structures. Here, corrections are required which are determined from epoch-by-epoch and satellite-by-satellite ionospheric modelling.

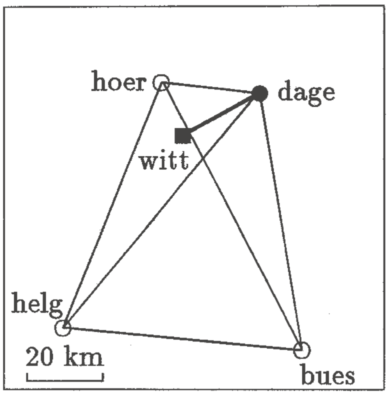


Figure 4a: Permanent GPS stations, baseline dage-witt (length: 23.1 km).

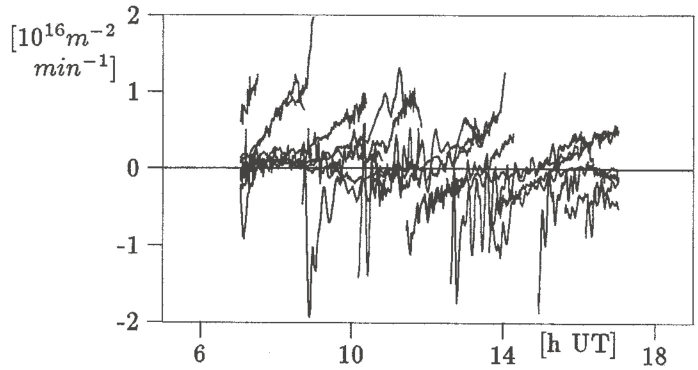


Figure 4b: Ionospheric conditions: Rate of TEC

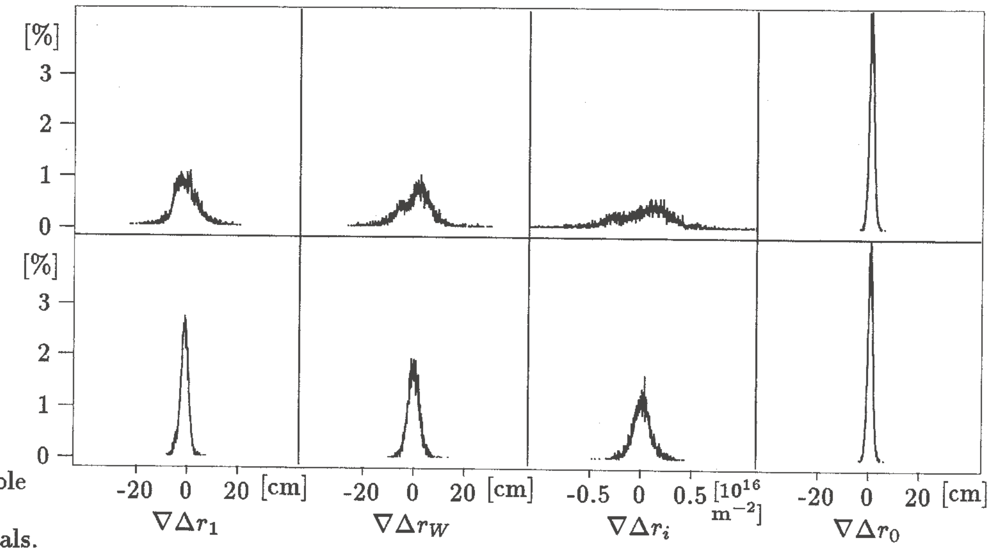


Figure 4c:
Distribution of double difference range (or free electron) residuals.

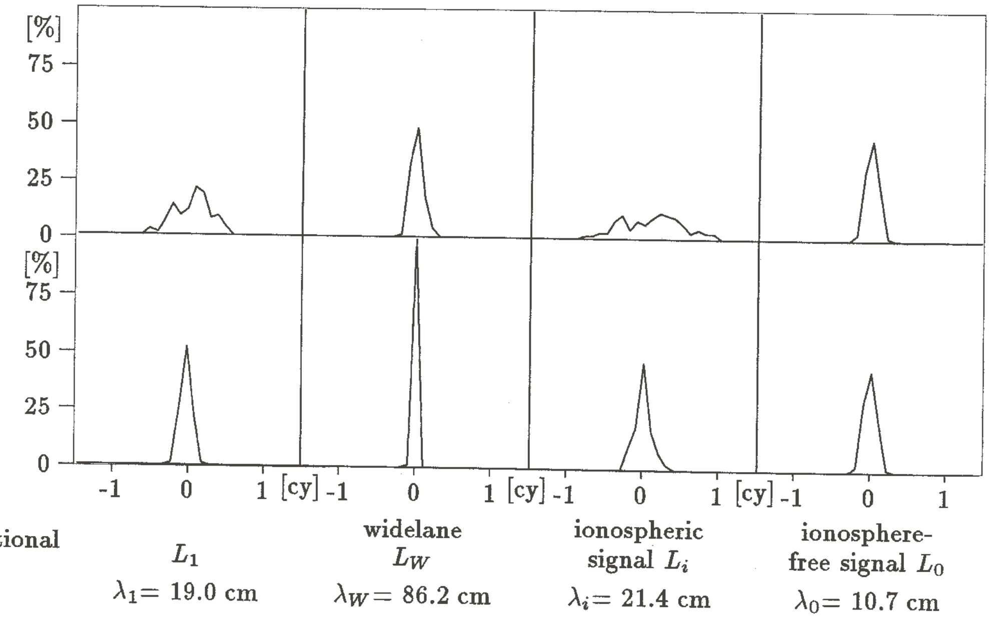


Figure 4d:
Distribution of fractional parts of ambiguities (15-min blocks of observations).

| Signal | original observations | with ionospheric corrections |
|---|---|---|
| $L_1$ | 2.5/4.0/5.0 | 1.1/0.7/2.4 |
| $L_0$ | 0.7/0.4/1.8 | 0.7/0.4/1.8 |

Table 2: Coordinate errors (RMS in cm) in latitude/longitude/height for observation periods of 15 minutes, baseline length 23.1 km, disturbed mid-latitude ionospheric conditions.

95% of the double difference ionosphere-free residuals of the 23.1 km baseline are smaller than $\pm 2.1$ cm (Figure 4c). Due to MSTIDs, ionospheric residuals are much larger ($\pm 0.65 \cdot 10^{16} m^{-2}$, 95%) than under undisturbed conditions. They cause errors in $L_1$-observations of $\pm 11.0$ cm (95%) and in widelane observations of $\pm 14.0$ cm (95%). These numbers are reduced to $\pm 3.5$ cm and $\pm 4.9$ cm respectively by applying ionospheric corrections.

Similar results were obtained from the distribution of the fractional parts of 15-minute double difference ambiguities (Figure 4d). Only 38% of the ionospheric signal's ambiguities were closer to the correct integers than 0.2 cycles. After the application of ionospheric corrections the percentage increased to 98. No difficulties in ambiguity resolution due to measurement errors were found for the widelane $L_W$ and for the ionosphere-free linear combination $L_0$. Fractional parts of $L_1$-ambiguities were successfully reduced in size by ionospheric corrections.

$L_1$ coordinate errors of 15-minute blocks of observations were reduced by ionospheric corrections from 2–4 cm to about 1 cm in latitude and longitude and from 5 cm to about half of it in height. They are now almost as accurate as the results from dual-frequency observations (Table 2). The epoch-by-epoch and satellite-by-satellite correction model removed most of the ionospheric effects of MSTIDs.

# 4 Conclusion

Distance-dependent errors due to ionospheric refraction complicate ambiguity resolution and limit the accuracy attainable in GPS baseline determination. Regional ionospheric error models are created epoch-by-epoch and satellite-by-satellite from double difference baseline residuals of a network of permanent stations. Ambiguity resolution in these long baselines is required to estimate and correct errors in medium-length baselines whose ambiguities can then be resolved with even shorter observation periods.

The requirement of dual-frequency ambiguity resolution between reference stations limits this approach to networks with baseline lengths of less than 50 to 100 km. It also prevents the (near-) real-time application of these differential GPS corrections.

Despite those limitations, these regional ionospheric error models enhance differential GPS in medium-sized networks. Most of the ionospheric effects are removed even under ionospheric disturbed conditions. Ambiguity resolution is improved for those techniques which rely on small ionospheric effects. Moreover, it was shown that the use of single-frequency receivers can be extended from short to medium-length baselines without a considerable loss of accuracy.

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